A test of the numerical solution of the complete equations for the case of free convection with heating from above indicates that the main properties of the flow structure determined by the shape of the temperature curve at the upper boundary are retained in a wide interval of Grashof numbers [2]. There is reason to expect the same thing for the analyzed case of mixed convection also.

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IRRIGATION AND AERATED-ZONE SOILS

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A constant specific flow rate q is used at the surface of the soil in irrigation and desalination; rather similar conditions occur at the soil surface during reservoir filling.

Consider the flow through a soil in the aeration zone for $q > k$ (k is the filtration factor); the region subject to the flushing then consists of two zones: a fully saturated one, where the pressure is $p > 0$ (the gravitational zone) and an incompletely saturated one, where the pressure is $p < 0$ (the capillary zone). Then a layer of water appears on the surface, whose depth increases with time. This situation has been examined for one particular case previously [1].

There are two approaches to such problems:

1. Vedernikov's theory indicates that the negative-pressure zone (capillary zone) has a negligible air content, while the capillary pressure $p_c = -\gamma H_c$ acts at the flooding front. Then for $q > k$ there are zones having positive and negative pressures, within which Laplace's equation applies.

2. Richards' nonlinear equation applies for the incompletely saturated zone; in that case, we have to consider the two zones together, with Laplace's equation applying to one and Richards' equation applying to the other.

Vedernikov's theory indicates that a supply rate $q \geq k$ causes the capillary negative pressure to vanish instantly at the surface of the soil, the result being a layer of water of depth H(t).

However, this results in a negative pressure at the surface of the soil covered by the water layer which is impossible; the conflict is easily eliminated by assuming that a water layer arises at the surface only after a certain time t_1, as is actually observed. The penetration into the soil occurs in two stages. In the first stage, there is no surface layer, while the capillary negative pressure falls from H_c to 0. In the second stage, the water layer appears, and the depth of this steadily increases.

Here we consider the penetration of the water into the soil on the assumption that H_c vanishes instantaneously; we also examine the effects of deleting this assumption. During the infiltration in the absence of the water layer, i.e., when the infiltrating water has not

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yet reached the groundwater level, the following balance equation applies for any time t:

$$qt = H(t) + \mu l(t) \qquad (q > k), \tag{1}$$

where l is the penetration depth, μ is the saturation deficiency, and $q = q_0 - \varepsilon$ is the difference between the input flow rate q_0 and the evaporation rate ε.

The differential equation for the penetration rate is then put as

$$\mu dl/dt = k[H(t) + l + H_c]/l. \tag{2}$$

The result from (1) and (2) in [2] was

$$\left[1 - \frac{l}{A_1 H_c (1+\bar{t})}\right]^{0.5+\alpha}\left[1 - \frac{l}{A_2 H_c (1+\bar{t})}\right]^{0.5-\alpha} = \frac{1}{1+\bar{t}}, \tag{3}$$

where

$$A_{1,2} = 0.5(k/q)(1/\mu - 1)(\alpha_0 \pm 1);\ \alpha_0 = \sqrt{1+\lambda};\ \alpha = 0.5/\alpha_0;$$
$$\lambda = 4(q/k)\mu/(1-\mu)^2;\ \bar{t} = qt/H_c$$

(the subscript 1 to A corresponds to the positive sign, while subscript 2 corresponds to the negative sign). Table 1 gives the values of α and $A_{1,2}$ for $\mu = 0.2$.

Then (3) gives a particular solution as $l = A_1 qt$ for $H_c = 0$.

We used (3) to construct $l/H_c = f_1(q/k, \bar{t})$ curves for $\mu = 0.2$ (Fig. 1a, dashed lines). These curves and (1) were used to define the depth of the water layer as a function of time $H/H_c = f_2(q/k, \bar{t})$ (Fig. 1b, dashed lines). It is clear from the behavior of these curves that $H < 0$ for a certain time t_0, and only for $t > t_0$ does H become positive. The solution to (3) for $q/k = 1$ gives a negative H for any value of $t (t_0 = \infty)$; negative H is impossible, and therefore joint solution of (1) and (3) gives a more or less correct result only for $t \geq t_0$.

Figure 2 shows $\bar{t}_0 = qt_0/H_c$ as a function of q/k for $\mu = 0.2$ (curve 1).

The values for t_0 and l_0 corresponding to $H = 0$ in Fig. 1 are determined by solving (1) and (3) for $H = 0$.

We differentiate (1) with respect to t, put $dH/dt = 0$ and substitute dl/dt from (2) into (1) to get

$$\frac{H_M}{H_c} = \frac{(q-k)\bar{t}_M - \mu k}{q - k(1-\mu)},\ \bar{t}_M = \frac{qt_M}{H_c}. \tag{4}$$

Then (1), (3), and (4) are solved together to define t_M, l_M, and H_M at point M, which corresponds to minimum H (Fig. 1); from (4) with $H_M = 0$ and $t_M = t_0$ we get $q/k = f(\mu, H_c)$, this being the result such that (3) applies for any t (in that case, negative H does not occur).

If H is to be positive for t small, or else zero, a different approach is required [3, 4].

We assume that during the period $0 < t < t_1$ the entire input flow q soaks into the soil, and therefore there is no surface layer ($H = 0$). At the start $t = 0$, the negative pressure H_c arises at the wetting front and at the surface of the soil. Disruption of the capillary forces at the surface causes this negative pressure to fall from H_c to zero over a period t_1. The negative pressure at the wetting front remains constant at H_c = const.

The boundary conditions are then as follows for this period: 1) the pressure at the infiltration front is $p(l, t) = -\gamma H_c$ = const; and 2) the flow rate is $q = -k\partial h(x, t)/\partial x$ = const.

The flow rate q is constant for any x and t, since in this case of one-dimensional infiltration the rate can only be a function of t and it becomes independent of t if the water is supplied at a constant rate.

The result q = const and specification of the pressure at the wetting front together ensure that the solution is unique; then the height of the equivalent capillary layer h_c at the surface is dependent on t, the value falling to zero from the maximum value H_c.

TABLE 1

q/k	1	2	3	4	6	8	10	12	14	15
α	0,333	0,267	0,229	0,204	0,172	0,151	0,136	0,125	0,116	0,112
A_1	5,00	2,87	2,13	1,75	1,30	1,08	0,93	0,83	0,76	0,72
A_2	1,00	0,87	0,78	0,72	0,63	0,58	0,53	0,50	0,47	0,46

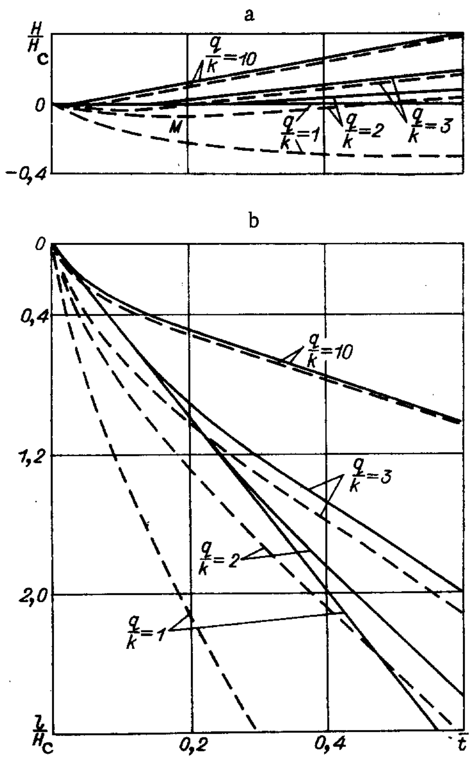

Fig. 1

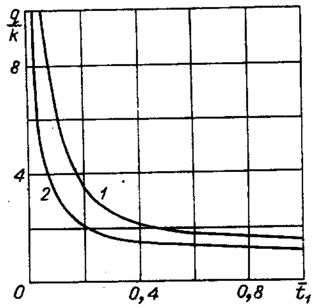

Fig. 2

From (1) and (2) with H = 0 we have

$$\mu dl/dt = k(-h_c + l + H_c)/l,\ qt = \mu l, \tag{5}$$

where h_c is the capillary negative pressure at the surface.

We then solve (5) with the conditions t = 0, l = 0, $h_c = H_c$ to get the wetted depth and the capillary vacuum as functions of time:

$$l = qt/\mu,\ h_c = H_c - (qt/\mu)(q/k - 1). \tag{6}$$

We find t_1 and l_1 (the wetted depth at the end of this period) from (6) with the conditions $t = t_1$, $l = l_1$, $h_c = 0$; we have

$$t_1 = \mu H_c k/q(q-k),\ l_1 = kH_c/(q-k). \tag{7}$$

Curve 2 of Fig. 2 shows $qt_1/H_c = \varphi(q/k)$ from $\mu = 0.2$.

In the subsequent period $t > t_1$, we derived the result from (1) by substituting H into (2) and integrating the equation from l, t to l_1, t_1, which gives

$$\left[\frac{A_1H_c(1+\bar{t})-l}{A_1H_c(1+\bar{t}_1)-l_1}\right]^{0.5+\alpha} = \left[\frac{A_2H_c(1+\bar{t}_1)+l_1}{A_2H_c(1+\bar{t})+l}\right]^{0.5-\alpha},\ \bar{t}_1 = \frac{qt_1}{H_c}. \tag{8}$$

The solid lines in Fig. 1a show $l/H_c = \varphi_1(\bar{t}, q/k)$, while those in Fig. 1b show $H/H_c = \varphi_2(\bar{t}, q/k)$, as constructed from (1) and (8) with $\mu = 0.2$.

The solution to (3) (broken lines in Fig. 1) may be compared with the results from (7) and (8) (solid lines in Fig. 1), which shows that (3) gives l larger but H smaller. The maximum difference between the results from (3) and (8) occurs for q/k = 1.0 but the difference then falls as q/k and t increase, and the difference does not exceed 10% for q/k ≥ 10.

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